

Supertubes

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It is shown that a IIA superstring carrying D0-brane charge can be ‘blown-up’, in a *Minkowski vacuum* background, to a $(1/4)$ -supersymmetric tubular D2-brane, supported against collapse by the angular momentum generated by crossed electric and magnetic Born-Infeld fields. This ‘supertube’ can be viewed as a worldvolume realization of the sigma-model Q-lump.

I. INTRODUCTION

M-theory can be viewed as an extension of string theory to a ‘democratic’ theory of branes, with dualities that allow the whole theory to be constructed from any brane. One aspect of this is that a given brane may often be considered as a partially collapsed version of another, higher-dimensional, brane. Conversely, there may be circumstances in which the lower-dimensional brane is ‘blown-up’ into the higher-dimensional one. One example of this is the observation that a IIA superstring can be blown-up to a tubular D2-brane by placing it in an appropriate (non-supersymmetric) background [1]; the background fields impose an external force that prevents the collapse of the D2-brane. Another example is the observation that a similar non-trivial background can blow-up a collection of D0-branes into a fuzzy 2-sphere [2]; this can again be considered as a D2-brane prevented from collapse by an external force [3]. Another way to support a brane against collapse to a lower-dimensional one is to allow it to carry angular momentum; the case of branes on spheres, first analyzed in [4], has recently found an application to ‘giant gravitons’ [5]; these partially preserve the supersymmetry of an $AdS_n \times S^m$ supergravity background [6], a fact that is presumably related to the appearance of angular momentum in the anticommutator of AdS -supersymmetry charges.

Here we shall show that a cylindrical, or tubular, D2-brane in a *Minkowski vacuum* spacetime can be supported against collapse by the angular momentum generated by crossed electric and magnetic Born-Infeld (BI) fields. The construction involves a uniform electric field along the tube, and a constant magnetic flux. The electric field can be interpreted as a ‘dissolved’ IIA superstring, so the tube is a IIA superstring that has been ‘blown-up’ to a tubular D2-brane. The magnetic flux can be interpreted as a ‘dissolved’ D0-brane charge (per unit length). There are thus similarities to some of the previously proposed stabilization mechanisms but also some differences. Firstly, the background in our case is trivial; there is *no external force*. Secondly, the tubular D2-brane configuration presented here is *supersymmetric*; it

preserves $1/4$ of the supersymmetry of the IIA Minkowski vacuum, hence the term ‘supertube’.

At first sight it would appear unlikely that a brane configuration stabilized by angular momentum could be supersymmetric. For a start, supersymmetry requires a time-independent energy density, which is certainly non-generic for configurations with non-zero angular momentum. Even if this can be arranged, there is still the fact that a supersymmetric configuration in Minkowski space minimizes the energy subject to fixed values of the central charges appearing in the supersymmetry algebra, but *angular momentum is not one of these charges*. However, although these considerations may make supersymmetric stabilization by angular momentum unlikely, they do not make it impossible. Indeed, in the somewhat different context of field theory solitons there are *known* examples of supersymmetric solitons that are stabilized by angular momentum. One such example is the Q -lump of certain massive $D = 3$ supersymmetric sigma-models [7], and the D2-brane supertube is essentially its *worldvolume* realization.

The Q -lump saturates an energy bound of the form $M = |L| + |Q|$, where L is a topological (lump) charge and Q is a Noether charge [8]. When viewed as a string-like solution of the maximally supersymmetric $D = 4$ sigma-model, it preserves $1/4$ of the supersymmetry of the sigma-model vacuum [9]. The simplest Q -lump-string has cylindrical symmetry [10], but for large Q its energy density is *not* concentrated on the axis of symmetry but rather in a hollow tube of radius Q . Therefore the Q -lump-string tube can be interpreted, for large Q , as a lump-string that has been ‘blown-up’ into a cylindrical tube of kink-2-brane [11]. This tube is prevented from collapse by a centrifugal force, generated by an angular momentum proportional to the charge Q . Because the effective action for the kink 2-brane is a Dirac-Born-Infeld (DBI) action [9], there should be a *worldvolume* description of the ‘blown-up’ sigma-model lump-string as a $1/4$ -supersymmetric solution of the $D = 4$ DBI equations. Any such solution will also solve the DBI equations of the $D = 10$ IIA D2-brane (in a IIA Minkowski vacuum) and must preserve at least two supersymmetries (this be-

ing 1/4 of the eight supersymmetries of the sigma-model vacuum). This solution is precisely the D2-brane ‘supertube’ to be discussed below; it actually preserves eight of the thirty-two supersymmetries of the IIA vacuum and is therefore still 1/4-supersymmetric.

II. ENERGETICS OF THE SUPERTUBE

The D2-brane Lagrangian, for unit surface tension, is

$$\mathcal{L} \equiv -\Delta = -\sqrt{\det(g+F)}, \quad (1)$$

where g is the induced worldvolume 3-metric and F is the BI 2-form field strength. We shall choose spacetime coordinates such that the $D = 10$ Minkowski metric is

$$ds^2 = -dT^2 + dX^2 + dR^2 + R^2 d\Phi^2 + ds^2(\mathbb{E}^6), \quad (2)$$

with $\Phi \sim \Phi + 1$. If we take the worldvolume coordinates to be (t, x, φ) with $\varphi \sim \varphi + 1$, then we may fix the worldvolume diffeomorphisms for a D2-brane of cylindrical topology by the ‘physical’ gauge choice

$$T = t, \quad X = x, \quad \Phi = \varphi. \quad (3)$$

For a cylindrical D2-brane of (possibly varying, but time-independent) radius $R(x, \varphi)$ at a fixed position in \mathbb{E}^6 (and with the X -axis as the axis of symmetry) the induced metric is

$$ds^2(g) = -dt^2 + dx^2 + R^2 d\varphi^2 + (R_x dx + R_\varphi d\varphi)^2, \quad (4)$$

where $R_x \equiv \partial_x R$ and $R_\varphi \equiv \partial_\varphi R$. We will allow for a time-independent electric field E in the x -direction, and a time-independent magnetic field B , so the BI 2-form field strength is

$$F = E dt \wedge dx + B dx \wedge d\varphi. \quad (5)$$

Under these conditions the Lagrangian becomes

$$\mathcal{L} = -\sqrt{(R^2 + R_\varphi^2)(1 - E^2) + B^2 + R^2 R_x^2}. \quad (6)$$

The corresponding Hamiltonian density is defined as

$$\mathcal{H} \equiv \Pi E - \mathcal{L}, \quad (7)$$

where $\Pi \equiv \partial \mathcal{L} / \partial E$ is the ‘electric displacement’ subject to the Gauss law constraint $\partial_x \Pi = 0$.

Let us now focus on the D2-brane supertube, for which R is constant. In this case, the relation between the electric field and the electric displacement takes the form

$$E = \frac{\Pi}{R} \sqrt{\frac{B^2 + R^2}{\Pi^2 + R^2}}, \quad (8)$$

so the Hamiltonian density becomes

$$\mathcal{H} = R^{-1} \sqrt{(\Pi^2 + R^2)(B^2 + R^2)}, \quad (9)$$

where it should be noted that $B = \nabla \times \mathbf{A}$ for BI 2-vector potential \mathbf{A} . The Gauss law constraint implies that Π is x -independent. In addition, the equation of motion for \mathbf{A} forces B to be x -independent when R is constant, as we are now assuming. Under these conditions \mathcal{H} is x -independent, and its integral over the circle parametrized by φ yields a constant energy per unit length, namely the tube tension

$$\tau = \oint d\varphi \mathcal{H}. \quad (10)$$

This is a function of R and a functional of $\Pi(\varphi)$ and $B(\varphi)$. For an appropriate choice of units, the integrals

$$q_s \equiv \oint d\varphi \Pi \quad \text{and} \quad q_0 \equiv \oint d\varphi B \quad (11)$$

are, respectively, the conserved IIA string charge and the D0-brane charge per unit length carried by the tube. The total D0-brane charge is also conserved, so imposing periodic boundary conditions on the tube, with period L , implies conservation of q_0 , and also that q_0 is quantized in multiples of some unit proportional to L^{-1} . For fixed values of these charges, the tube tension is bounded from below:

$$\tau \geq |q_s| + |q_0|, \quad (12)$$

with equality if and only if

$$\Pi = q_s, \quad B = q_0, \quad R = \sqrt{|q_s q_0|}. \quad (13)$$

The crossed electric and magnetic fields generate a Poynting 2-vector-density with

$$\mathcal{P}_\varphi = \Pi B \quad (14)$$

as its only non-zero component. The integral of \mathcal{P}_φ over φ yields an angular momentum per unit length $J = \Pi B = q_s q_0$ along the axis of the cylinder. It is this angular momentum that supports the tube at the constant radius $\sqrt{|q_s q_0|}$. Substituting (13) into (8) we see that

$$E = \text{sgn}(\Pi) = \pm 1. \quad (15)$$

This would be the ‘critical’ electric field if the magnetic field were absent, as is shown by the fact that $\Delta = B$ when $|E| = 1$.

III. SUPERSYMMETRY

We now aim to show that the tubular D2-brane configuration just described is 1/4-supersymmetric; but as we also aim to relate it to some previously discussed D2-brane configurations we shall now drop the assumption that the worldvolume fields are independent of x and φ . The number of supersymmetries preserved by any D2-brane configuration is the number of independent Killing spinors ϵ of the background for which

$$\Gamma \epsilon = \epsilon, \quad (16)$$

where Γ is the matrix appearing in the ‘ κ -symmetry’ transformation of the worldvolume spinors [12]. Introducing $\Gamma_{\mathfrak{h}}$ as the constant matrix with unit square which anticommutes with all ten spacetime Dirac matrices, and $(\gamma_t, \gamma_x, \gamma_\varphi)$ as the induced worldvolume Dirac matrices, we have [13]

$$\Gamma = \Delta^{-1} (\gamma_{tx\varphi} + E \gamma_\varphi \Gamma_{\mathfrak{h}} + B \gamma_t \Gamma_{\mathfrak{h}}). \quad (17)$$

For the D2-brane configuration of interest here

$$\begin{aligned} \gamma_t &= \Gamma_{\underline{T}}, & \gamma_x &= \Gamma_{\underline{X}} + R_x \Gamma_{\underline{R}}, \\ \gamma_\varphi &= R \Gamma_{\underline{\Phi}} + R_\varphi \Gamma_{\underline{R}}, \end{aligned} \quad (18)$$

where $\Gamma_{\underline{T}}, \Gamma_{\underline{X}}, \Gamma_{\underline{R}}$ and $\Gamma_{\underline{\Phi}}$ are the constant Minkowski spacetime Dirac matrices (with $\Gamma_{\underline{\Phi}}^2 = 1$). For the space-time coordinates that we have chosen, any Killing spinor ϵ can be written as

$$\epsilon = M_+ \epsilon_0, \quad M_\pm \equiv \exp\left(\pm \frac{1}{2} \Phi \Gamma_{\underline{R}\underline{\Phi}}\right), \quad (19)$$

where ϵ_0 is a constant 32-component spinor of $Spin(1, 9)$. The condition for preservation of supersymmetry can now be written as

$$\begin{aligned} 0 &= M_+ (RR_x \Gamma_{\underline{T}R\Phi} + B \Gamma_{\underline{T}} \Gamma_{\mathfrak{h}} + R_x R_\varphi \Gamma_{\underline{T}} - \Delta) \epsilon_0 \\ &\quad + M_- \gamma_\varphi \Gamma_{\mathfrak{h}} (\Gamma_{\underline{T}X} \Gamma_{\mathfrak{h}} + E) \epsilon_0. \end{aligned} \quad (20)$$

It is clear that in order to satisfy this equation for all values of φ (equal to Φ for our gauge choice) both terms on the right hand side must vanish independently. From the vanishing of the second term we recover the condition that $E = \pm 1$, and we further deduce that ϵ_0 must satisfy

$$\Gamma_{\underline{T}X} \Gamma_{\mathfrak{h}} \epsilon_0 = -\text{sgn}(E) \epsilon_0. \quad (21)$$

The vanishing of the first term leads to $R_\varphi = 0$ and

$$(RR_x \Gamma_{\underline{T}R\Phi} + B \Gamma_{\underline{T}} \Gamma_{\mathfrak{h}}) \epsilon_0 = \sqrt{R^2 R_x^2 + B^2} \epsilon_0. \quad (22)$$

For the supertube configuration $R_x = 0$ and B is constant, so the constraint above becomes simply

$$\Gamma_{\underline{T}} \Gamma_{\mathfrak{h}} \epsilon_0 = \text{sgn}(B) \epsilon_0. \quad (23)$$

The two conditions (21) and (23) are compatible and imply preservation of 1/4 supersymmetry; the minimal energy tubular D2-brane configuration is a *supertube*. Note that the constraints (21) and (23) are those associated with, respectively, a IIA superstring (in the X -direction) and D0-brane charge; in particular, there is no trace of the D2-brane in these conditions! The physical reason for this is that a cylindrical D2-brane carries no net D2-brane charge.

When $R_x \neq 0$, the vanishing of the first term on the right hand side of (20) implies that

$$B = B_0 R R_x \quad (24)$$

for some constant B_0 , and the constraint on the spinor becomes

$$(\Gamma_{\underline{T}R\Phi} + B_0 \Gamma_{\underline{T}} \Gamma_{\mathfrak{h}}) \epsilon_0 = \sqrt{1 + B_0^2} \epsilon_0. \quad (25)$$

The Gauss law now implies that

$$R(x) = C e^{x/E_0} \quad (26)$$

for some constants C and E_0 . This is just the ‘BIon in a magnetic background’ solution of [9] representing a IIA string ending on a bound state of D2-branes and D0-branes. To see this more clearly, first note that the constraint (25) indeed corresponds to that of a D2-D0 bound state in the (R, Φ) -plane. Secondly, we can invert (26) to find

$$x = E_0 \ln(R/C), \quad (27)$$

which shows that x is a harmonic function on the D2-brane 2-dimensional worldspace, as expected for the BIon [14,15]. Finally, using (27) we can rewrite the BI field strength (5) as

$$F = \frac{E_0}{R} dt \wedge dR + B_0 R dR \wedge d\varphi. \quad (28)$$

This corresponds to a radial Coulomb-like electric field on the D2-brane worldspace, as expected from the charge at the endpoint of the string, and to a constant density of D0-brane charge per unit worldspace area, precisely as in the solution of [9].

IV. DISCUSSION

By definition, a Dp-brane is a sink for IIA string charge. However, the D0-brane, being point-like, is special because the IIA charge has nowhere to go and must exit on another IIA superstring. Thus D0-branes can appear as ‘beads’ on a IIA superstring, breaking the 1/2 supersymmetry of the string to 1/4 supersymmetry. Of course, quantum mechanics will ensure that the ground state of such a superstring is one for which the D0-brane charge is uniformly distributed along the string. This ‘charged’ IIA superstring will have a tension exactly equal to the D2-brane supertube discussed above, $\tau = |q_s| + |q_0|$. What distinguishes them is the angular momentum; the charged superstring has zero angular momentum while the supertube has angular momentum per unit length J equal to $q_s q_0$. In principle, J can be specified *independently* of the string and D0-brane charges, so given q_s and q_0 we might expect there to be some supersymmetric string/tube configuration with arbitrary J . As long as $|J| \leq |q_s q_0|$ it is not difficult to see what this will be: a supertube with angular momentum per unit length J , together with a charged superstring along

its central axis. Because J is quantized in the same units as B , it is always possible for excess string and D0-charge to ‘condense’ out of a tubular D2-brane to leave behind a supertube supported from collapse by any given $|J|$ less than $|q_s q_0|$. On the other hand, it is unclear what the ground state could be when $|J| > |q_s q_0|$. It is conceivable that $|q_s q_0|$ is an upper bound on the angular momentum of a *supersymmetric* IIA superstring configuration with charges q_s and q_0 , and that supersymmetry is spontaneously broken when $|J| > |q_s q_0|$.

There is some similarity here to the status of angular momentum in the context of black holes of $D = 5$ supergravity [16–18]. The black hole mass is determined entirely by its charge, if it is supersymmetric. For a given charge there is a one-parameter family of supersymmetric black hole spacetimes, parametrized by an angular momentum, but there is a critical value of the angular momentum beyond which the physics is qualitatively different. There is also a similarity to the suggestion [6] that the ‘giant graviton’ ground state is not supersymmetric above a critical value of the angular momentum, although the supersymmetry of relevance in that case is *AdS* rather than Poincaré.

Finally, we wish to comment on some M-theory configurations dual to the D2-brane supertube. The first one involves the M2-brane: the D2-brane action is equivalent to the action for the $D = 11$ supermembrane, the equivalence involving an exchange of the BI 1-form potential for a periodically-identified scalar field Ψ (with unit period) representing position in the 11th dimension [13]. Specifically, one has

$$\partial_i \Psi = \frac{g_{ij} \varepsilon^{jkl} F_{kl}}{2\sqrt{-\det(g + F)}}. \quad (29)$$

For the D2-brane supertube this yields

$$\Psi = \Pi \varphi - t. \quad (30)$$

This implies, firstly, that the M2-brane is wound Π times around the 11th dimension, as expected given the identification of Π with IIA string charge. Secondly, it implies that Π must be an integer, obviously equal to the number of IIA strings dissolved in the original D2-brane. Thirdly, it implies that there is a wave at the speed of light in the 11th dimension, as expected from the D0-brane charge; the momentum of this wave is proportional to B . Note that if the x dimension is periodically identified with period L then one can take $L \rightarrow 0$ to arrive at a new (1/4)-supersymmetric IIA configuration in which a helical string rotates, producing a net momentum along the axis of the helix [19].

Another dual M-theory configuration consists of an M5-brane with topology $\mathbb{R}^4 \times S^1$, carrying dissolved membrane charges oriented along orthogonal planes in \mathbb{R}^4 . To see this, let the S^1 be parametrized by φ , and let the two planes span the directions 1-2 and 3-4. Reducing to the IIA theory by compactifying the 4-direction, and T-dualizing along 2 and 3 leads to the D2-brane supertube.

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- [1] R. Emparan, *Born-Infeld strings tunneling to D-branes*, Phys. Lett. **423B** (1988) 71.
 - [2] R.C. Myers, *Dielectric branes*, JHEP 9912 (1999) 022.
 - [3] J. Pawelczyk and S.-J. Rey, *Ramond-Ramond flux stabilization of D-branes*, Phys.Lett. **493B** (2000) 395.
 - [4] J. Hoppe and H. Nicolai, *Relativistic minimal surfaces*, Phys. Lett. **196B** (1987) 451.
 - [5] J. McGreevy, L. Susskind and N. Toumbas, *Invasion of the giant gravitons from anti-de Sitter space*, JHEP 0006 (2000) 008.
 - [6] M.T. Grisaru, R.C. Myers and O. Tafjord, *SUSY and Goliath*, JHEP 0008 (2000) 040.
 - [7] R. Leese, *Q-lumps and their interactions*, Nucl. Phys. **B366** (1991) 283.
 - [8] E.R.C. Abraham, *Non-linear sigma models and their Q-lump solutions*, Phys. Lett. **278B** (1992) 291.
 - [9] J.P. Gauntlett, R. Portugues, D. Tong and P.K. Townsend, *D-brane solitons in supersymmetric sigma-models*, Phys. Rev. **D63** (2001) 085002.
 - [10] This is an oversimplification because the integral of the charge density of the cylindrically-symmetric solution has an IR logarithmic divergence; for this reason the ‘charge’ Q mentioned below should be regarded only as a measure of scale.
 - [11] E.R.C. Abraham and P.K. Townsend, *More on Q-kinks: a (1+1)-dimensional analogue of dyons*, Phys. Lett. **295B** (1992) 225.
 - [12] See, for example, E. Bergshoeff, R. Kallosh, T. Ortín and G. Papadopoulos, *Kappa-symmetry, supersymmetry and intersecting branes*, Nucl. Phys. **B502** (1997) 149.
 - [13] See, for example, E. Bergshoeff and P.K. Townsend, *Super D-branes*, Nucl. Phys. **B490** (1997) 145.
 - [14] C.G. Callan and J.M. Maldacena, *Brane dynamics from the Born-Infeld action*, Nucl. Phys. **B513** (1998) 198.
 - [15] G.W. Gibbons, *Born-Infeld particles and Dirichlet p-branes*, Nucl. Phys. **B514** (1998) 603.
 - [16] J.C. Breckenridge, D.A. Lowe, R.C. Myers, A.W. Peet, A. Strominger and C. Vafa, *Macroscopic and microscopic entropy of near-extremal spinning black holes*, Phys. Lett. **381B** (1996) 423.
 - [17] J.P. Gauntlett, R.C. Myers and P.K. Townsend, *Black holes of $D = 5$ supergravity*, Class. Quantum Grav. **16** (1999) 1.
 - [18] G.W. Gibbons and C.A.R. Herdeiro, *Supersymmetric rotating black holes and causality violation*, Class. Quantum Grav. **16** (1999) 3619.
 - [19] We owe this observation to Barak Kol and Juan Maldacena; note that T-duality along the axis of the helix interchanges the string with the wave yielding a IIB version of the rotating helical string.